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Cross-Currents

A great amount of data would probably be needed for a satisfactory explanation of the following phenomenon. The American secondary schools are with apparent uniformity diminishing the amount of mathematics required of their students in seeming disregard of the facts that:

(1) An increased use of mathematics is steadily being called for in the fields accustomed to use it, as physics, chemistry, engineering,

(2) New fields are steadily adopting it as an indispensable tool, as statistics, economics, biology, and certain branches of educational theory.

Here indeed is a case of cross-currency whose existence may with difficulty be explained. One thing only seems fairly certain: Objectives in the modern high school program are being and have been set up by secondary school administrations with frank omission of consideration for the more remote reaches of intellect, culture, science and research. Without presuming to pronounce upon the wisdom of the omission we may yet legitimately call attention to some consequences of this co-existence of two unblended school systems, one aiming to prepare the individual for citizenship and a vocation, the other aiming to develop in him the highest degree of intellectual culture and service possibilities.

(a) Probably one of the most important consequences of the continuation of this cross-currency of secondary and higher school programs will be that ultimately the colleges will be forced to take charge of the beginning courses in algebra and geometry, which heretofore have been embraced in the secondary curricula. The effect of such change in the college organization would be that either the collegiate training period would have to be increased or undesirable abbreviations of important course material would have to be made. Either consequence would have questionable value.

(b) It would appear that avoidance of the condition noted in (a) could only be brought about by some kind of compromise between college administrations and college mathematics departments by virtue of which the mathematics needed by the student in physics, chemistry and engineering would be administered to him almost entirely in formulas cut-and-dried and taken on faith. Such substitution of mechanized mathematics for mathematics made rational would in the end lower ideals in both research and culture programs.

(c) Lamentable from the point of view of those who believe that mathematics is ideal for developing the individual's power to analyze—would be the establishment of secondary school policies that result in minimizing, if not altogether eliminating, courses primarily administered for mental discipline's sake. The misfortune resulting from such policies would be multiplied and deepened by the fact, universally received, that if intellectual disciplines are real in any sense their reality for the unfolding mind must be greater throughout its high school period than throughout any of its later periods of development.

(d) More subtle than the type of consequence noted in (c) but nevertheless fearfully subversive in its effects upon the entire mathematical substructure of our civilization and culture would be that state of the American public mind which would certainly ensue if everywhere some time it should be heralded that all school administrations of the land had made the study of mathematics optional with the student. Intelligence or unintelligence of the option to be exercised by him would then have but small weight as against the unmeasured weight of a united American public opinion that had tacitly approved its virtual displacement from secondary study programs.

(e) From one generation to the next, however wide may be the swing of the pendulum of practice respecting mathematics in the American secondary schools, the destiny of this glorious science must forever remain unaffected. In every age of the world not one mathematical genius alone but many such will continue to be born. Centuries to come will bear witness to times when mathematics increasingly will become the handmaid not alone of physics and engineering but of the social, the civic and the moral sciences, when the "universal art apodeictic" will be made the corner-stone of all organized thought having for objective the solution of every problem vital to human development and to human welfare.

—S. T. S.

*To Construct a Magic Square of Order 2n From a Given Square of Order n**

By A. L. CANDY
University of Nebraska

Magic Squares are classified as odd or even according as n, the number of cells in a row, is odd or even. An even square is said to be Evenly Even when $\frac{1}{2}n$ is an even number, and Oddly Even when $\frac{1}{2}n$ is an odd number. A square is said to be Perfect, or Symmetric with respect to the Center, when the sum of any two numbers which are symmetric with the center is equal to the sum of the first and last numbers in the series.

There are a number of well known and comparatively simple methods of constructing either perfect or imperfect odd squares, or evenly even squares. But the methods of constructing any oddly even squares are much more complicated. In fact there seems to be no well known method of constructing an oddly even square that shall be completely symmetrical. The Method of Current Groups here presented seems therefore to be of especial interest, because it furnishes a comparatively easy way of constructing an oddly even square from any given odd square, and if the given odd square is symmetric, the oddly even square can all be made symmetric except two pairs of numbers, and these two pairs will be symmetric with respect to the horizontal or vertical axis. If the given square is even and perfect, it is very easy to secure complete symmetry in the resulting square.

In order to make the process—which is very simple—more conspicuous, heavy lines are used in all the following figures to form the cells of the given square, and the small numbers, put in light italic figures at the center of these large cells form the given magic square of order n. Now divide each of these large cells into four small cells thus giving $4n^2$ cells for the required square of order $2n$. Then in the four cells thus formed within the large cell that contains the number 1, write the first four consecutive numbers 1,2,3,4, and in the four cells within the large cell containing the number 2 write the second four

*This subject is mentioned briefly in the article on Magic Squares by F. A. P. Barnard in Johnson's Cyclopedia. But the method is not very clearly explained and its full possibilities are not even suggested. It is called the method of *Current Groups*.

John Willis devotes Chap. VII of his book on Magic Squares to the same subject. He calls the groups "Quartettes." His method of selecting these quartettes is not very easy to follow. Besides he makes no attempt to secure symmetric squares.

5	7	35	36	15	13
	2		9		4
8	6	34	33	14	16
25	27	19	20	11	9
	7		5		3
28	26	17	18	10	12
21	23	4	3	31	29
	6		1		8
24	22	2	1	30	32

FIG. 1.
Com. Dif. $=1$

4	21	35	36	8	7
	2		9		4
22	3	18	17	25	26
31	14	27	28	5	6
	7		5		3
13	32	9	10	23	24
11	12	20	19	34	15
	6		1		8
30	29	2	1	16	33

FIG. 4.
$d_1 = 1 \quad d_2 = 18 = 2n^2$

13	19	33	36	8	2
	2		9		4
22	16	30	27	5	11
3	9	20	23	31	25
	7		5		3
12	6	14	17	28	34
26	32	10	7	21	15
	6		1		8
35	29	4	1	18	24

FIG. 2.
Com. Dif. $= n = 3$.

4	9	35	36	14	13
	2		9		4
10	3	30	29	19	20
31	26	21	22	5	6
	7		5		3
25	32	15	16	11	12
17	18	8	7	34	27
	6		1		8
24	23	2	1	28	33

FIG. 5.
$d_1 = 1 \quad d_2 = 6 = 2n$

2	20	27	36	22	4
	2		9		4
29	11	18	9	13	31
7	25	23	32	21	3
	7		5		3
34	16	5	14	12	30
6	24	28	19	26	8
	6		1		8
33	15	10	1	17	35

FIG. 3.
Com. Dif. $= n^2 = 9$

5	20	33	36	10	7
	2		9		4
23	2	18	15	25	28
31	16	26	29	3	6
	7		5		3
13	34	8	11	21	24
9	12	22	19	35	14
	6		1		8
30	27	4	1	17	32

FIG. 6.
$d_1 = 3 = n \quad d^2 = 18 = 2n^2$

consecutive numbers 5,6,7,8, in large cell No. 3 write the third consecutive group 9,10,11,12. Proceed in like manner until all the cells are filled. The squares in Figs. 1, 7, 13, 22, have all been constructed by selecting the consecutive groups in this manner.

29	30	16	14	99	100	62	64	46	45
	8		4		25		16		12
32	31	13	15	98	97	63	61	47	48
5	6	92	90	75	76	58	60	22	21
	2		23		19		15		6
8	7	89	91	74	73	59	57	23	24
81	82	68	66	51	52	34	36	18	17
	21		17		13		9		5
84	83	65	67	49	50	35	33	19	20
77	78	44	42	28	27	10	12	94	93
	20		11		7		3		24
80	79	41	43	25	26	11	9	95	96
53	54	40	38	4	3	86	88	70	69
	14		10		1		22		18
56	55	37	39	2	1	87	85	71	72

FIG. 7. Com. Dif. = 1

But these groups can not be written in these 2x2 squares in any order at random. Here lies the only difficulty in the process. Particular care must be given to the order in which these groups are written, especially if the desired square is to be perfect. There is, however, no necessary order for any group or combination of groups. For this reason it is possible to construct many squares from any given square and with any given set of groups. Four numbers in a 2x2 square may be written in three essentially different orders, which may be called the Cyclic Order $\begin{matrix}1&2\\4&3\end{matrix}$, the Crossed Order $\begin{matrix}1&3\\4&2\end{matrix}$, and the Z order $\begin{matrix}1&2\\3&4\end{matrix}$, where the figures 1, 2, 3, 4, indicate the order of the numbers in the group. Each of these may be reversed,—columns interchanged—, or inverted—rows interchanged. Thus we have the following twelve possible orders:

1 2 4 3 2 1 3 4 1 3 4 2 3 1 2 4 1 2
4 3, 1 2, 3 4, 2 1, 4 2, 1 3, 2 4, 3 1, 3 4,

3 4 2 1 4 3
1 2, 4 3, 2 1,

Now we observe that if any group of four consecutive numbers be written in any one of the cyclic orders, or in any crossed order, the sum of the two numbers in each column will be the same, the difference between the sums of the two rows will be four in any cyclic order, and two in any crossed order. If written in any Z order the sums of the diagonals will be the same, the difference of rows will be four, and the difference of the columns will be two.

42	47	76	66	95	100	9	19	28	23
	8		4		25		16		12
57	52	61	71	90	85	14	4	33	38
21	26	60	50	74	79	88	98	7	2
	2		23		19		15		6
36	31	45	55	69	64	93	83	12	17
5	10	39	29	53	58	67	77	86	81
	21		17		13		9		5
20	15	24	34	43	48	72	62	91	96
84	89	18	8	37	32	46	56	70	65
	20		11		7		3		24
99	94	3	13	22	27	51	41	75	80
63	68	97	87	16	11	30	40	49	44
	14		10		1		22		18
78	73	82	92	6	1	35	25	54	59

FIG. 8. Com. Dif $=n=5$

Therefore if all the groups be written in cyclic or crossed orders exclusively the resulting square will be magic as to the columns. Hence it will be sufficient to choose the orders so as to balance the rows. In the figures where n is even, only cyclic and crossed orders have been used. In all cases where n is odd (except figures 4, 5, 6,

10, 11, which are somewhat irregular) two groups only have been written in the Z orders. The reason for using these two Z orders will be explained later. Therefore the rows may be balanced as follows: If the given square is even, the rows may be balanced by writing for every group in cyclic order another group in the inverse cyclic order. Likewise with groups in the crossed orders. Or a single group in cyclic order may be balanced by two crossed groups both in the inverse order. When the given square is odd this must be done, for there is one group that cannot be paired. This group, preferably the

11	36	99	49	57	82	45	95	28	3
	11		24		7		20		3
86	61	24	74	32	7	70	20	53	78
4	29	87	37	75	100	33	83	41	16
	4		12		25		8		16
79	54	12	62	50	25	58	8	66	91
17	42	80	30	63	88	46	96	34	9
	17		5		13		21		9
92	67	5	55	13	38	71	21	59	84
10	35	93	43	76	51	39	89	47	22
	10		18		1		11		22
85	60	18	68	1	26	64	14	72	97
23	48	81	31	94	69	27	77	40	15
	23		6		19		2		15
98	73	6	56	44	19	52	2	65	90

FIG. 9 Com. Dif. $= n^2 = 25$

middle one, must be written in cyclic order with third and fourth numbers in the upper row, then two other groups must be written in the inverse crossed order, that is, second and fourth numbers in the lower row. (These three groups may all be inverted). Then the remainder of the row, an even number of groups, may be balanced as in the even squares. If the diagonals of the required square do not now have the correct sum, this defect may be remedied by reversing the order of some of the groups in the diagonals of the given square. Reversing cyclic or crossed groups will not affect the balance of either

the rows or columns of the required square. If the Z orders are introduced, at least two in reverse order must be used in any column in order to balance the two columns of the required square.

If the given square is symmetric, the required square may be made symmetric, otherwise not. But this will call for much more care in arranging the order of the groups. If the pairs of groups in pairs of symmetric cells of the square of order n are respectively symmetric, the square of order $2n$ will be completely symmetric. It should be noted, however, that a symmetric square is not necessarily

15	16	7	58	99	100	31	32	73	74
	8		4		25		16		12
66	65	57	8	50	49	82	81	24	23
3	4	46	95	87	88	29	30	61	62
	2		23		19		15		6
54	53	96	45	38	37	80	79	11	12
42	91	33	84	75	76	17	18	60	9
	21		17		13		9		5
92	41	83	34	25	26	68	67	10	59
89	90	22	21	64	63	56	5	48	47
	20		11		7		3		24
39	40	71	72	13	14	6	55	97	98
78	77	20	19	52	51	93	44	36	35
	14		10		1		22		18
27	28	70	69	2	1	43	94	85	86

FIG. 10. $d_1 = 1 \quad d_2 = 50 = 2n^2$

magic. If two such symmetric groups are written in reversed cyclic order, or reversed crossed order, the two groups will be symmetric and balanced as to columns but not as to rows. Hence one-half of these pairs of symmetric groups must be written in inverse order with respect to the other half in order to balance the rows.

The easiest way therefore to secure a symmetric square is as follows: Write all the groups that fill the cells of the same column of the given square in the same order (cyclic say), and write all the groups in the symmetric column in the same order only reversed.

Treat two other symmetric columns in the same way, but have the groups all inverted with respect to the groups in the other two columns. In this manner complete symmetry may be secured provided the given square is evenly even.

If the given square is oddly even all columns except two can be made symmetric as in the case of the evenly even square. The groups in these two columns (the two in the center in accompanying figures.) must be both symmetric and balanced as to columns. Hence groups in the same horizontal row must be in inverted order to balance rows.

25	26	7	8	89	100	72	71	54	53
	8		4		25		16		12
36	35	18	17	99	90	61	62	43	44
3	4	86	95	77	78	50	49	32	31
	2		23		19		15		6
14	13	96	85	68	67	59	60	21	22
81	91	73	64	55	56	27	28	10	20
	21		17		13		9		5
92	82	63	74	45	46	37	38	19	9
79	80	41	42	34	33	16	5	88	87
	20		11		7		3		24
70	69	52	51	24	23	6	15	97	98
57	58	39	40	2	11	84	83	66	65
	11		10		1		22		18
48	47	30	29	12	1	93	94	75	76

FIG. 11. $d_2 = 1 \quad d_1 = 10 = 2n$

Since groups in inverted order can not be symmetric, groups in the upper half of these columns must be inverted with respect to those of the lower half of the same column, and reversed with respect to those in the other column. Or, since four groups in the same crossed order can be balanced as to rows by two groups of the same cyclic order, two pairs of symmetric columns can be made symmetric by using all crossed groups without any inversions, and these can be balanced as to rows by a single pair of symmetric columns with groups in cyclic order.

These order schemes may be indicated in the following manner: For evenly even squares

1	2	4	3	3	4	2	1		1	3	4	2	2	4	3	1
4	3,	1	2,	2	1,	3	4	or	4	2,	1	3,	3	1,	2	4

The latter was used in Fig. 15, the former, in all other cases. For oddly even squares

(1)

	1	2	4	3	1	2	3	4	3	4	2	1
Upper half	4	3,	1	2,	4	3,	2	1,	2	1,	3	4
Lower half	1	2	4	3	4	3	2	1	3	4	2	1
	4	3,	1	2,	1	2,	3	4,	2	1,	3	4

or

(2)

1	3	3	1	4	3	3	4	1	3	3	1
4	2,	2	4,	1	2,	2	1,	4	2,	2	4

13	18	4	9	45	100	86	81	77	72
	8		4		25		16		12
68	63	59	54	95	50	31	36	22	27
2	7	48	93	84	89	30	25	66	61
	2		23		19		15		6
57	52	98	43	39	34	75	80	11	16
41	91	82	37	73	78	14	19	10	60
	21		17		13		9		5
96	46	32	87	23	28	64	69	55	5
85	90	21	26	67	62	58	3	49	44
	20		11		7		3		24
40	35	76	71	17	12	8	53	94	99
74	79	65	70	6	51	47	42	38	33
	14		10		1		22		18
29	24	20	15	56	1	92	97	83	88

FIG. 12. $d_1 = 5 = n \quad d_2 = 50 = 2n^2$

The first method was used in Figs. 18 and 22.

If the given square is odd the center column is self-symmetric. Hence the groups above the center cell may be written $\begin{matrix}3 & 4\\ 2 & 1,\end{matrix}$ those below the center cell $\begin{matrix}4 & 3\\ 1 & 2.\end{matrix}$ Then in order to make the groups in the three center columns symmetric and balanced as to rows the following orders must be used:

Above center cell $\begin{matrix}1 & 3\\ 4 & 2,\end{matrix}$ $\begin{matrix}3 & 4\\ 2 & 1,\end{matrix}$ $\begin{matrix}3 & 1\\ 2 & 4,\end{matrix}$ below center cell $\begin{matrix}1 & 3\\ 4 & 2,\end{matrix}$ $\begin{matrix}4 & 3\\ 1 & 2,\end{matrix}$ $\begin{matrix}3 & 1\\ 2 & 4.\end{matrix}$

(*Discussion Continued on Page 157*)

69	70	48	47	21	23	195	196	143	141	119	120	94	93
18		*12*		*6*		*49*		*36*		*30*		*24*	
72	71	45	46	24	22	194	193	142	144	118	117	95	96
37	38	16	15	185	187	163	164	139	137	87	88	62	61
10		*4*		*47*		*41*		*35*		*22*		*16*	
40	39	13	14	188	186	162	161	138	140	86	85	63	64
5	6	180	179	153	155	131	132	107	105	83	84	30	29
2		*45*		*39*		*33*		*27*		*21*		*8*	
8	7	177	178	156	154	130	129	106	108	82	81	31	32
169	170	148	147	121	123	99	100	75	73	51	52	26	25
43		*37*		*31*		*25*		*19*		*13*		*7*	
172	171	145	146	124	122	97	98	74	76	50	49	27	28
165	166	116	115	89	91	68	67	43	41	19	20	190	189
42		*29*		*23*		*17*		*11*		*5*		*48*	
168	167	113	114	92	90	65	66	42	44	18	17	191	192
133	134	112	111	57	59	36	35	11	9	183	184	158	157
34		*28*		*15*		*9*		*3*		*46*		*40*	
136	135	109	110	60	58	33	34	10	12	182	181	159	160
101	102	80	79	53	55	4	3	175	173	151	152	126	125
26		*20*		*14*		*1*		*44*		*38*		*32*	
104	103	77	78	56	54	2	1	174	176	150	149	127	128

FIG. 13 d=1 14x14

1	3	56	54	46	48	27	25
	1		15		14		4
7	5	50	52	44	42	29	31
26	28	47	45	53	55	4	2
	12		6		7		9
32	30	41	43	51	49	6	8
57	59	16	14	22	24	35	33
	8		10		11		5
63	61	10	12	20	18	37	39
34	36	23	21	13	15	60	58
	13		3		2		16
40	38	17	19	11	9	62	64

FIG. 14. $d=2$

1	9	32	24	55	63	42	34
	1		15		14		4
25	17	8	16	47	39	50	58
38	46	59	51	20	28	13	5
	12		6		7		9
62	54	35	43	12	4	21	29
36	44	61	53	22	30	11	3
	8		10		11		5
60	52	37	45	14	6	19	27
7	15	26	18	49	57	48	40
	13		3		2		16
31	23	2	10	41	33	56	64

FIG. 16. $d=2n=8$

1	5	59	63	52	56	10	14
	1		15		14		4
7	3	61	57	54	50	16	12
46	42	24	20	31	27	37	33
	12		6		7		9
44	48	18	22	25	29	35	39
26	30	36	40	43	47	17	21
	8		10		11		5
32	28	38	34	45	41	23	19
53	49	15	11	8	4	62	58
	13		3		2		16
51	55	9	13	2	6	60	64

FIG. 15. $d=2$

1	32	50	47	46	51	29	4
	1		15		14		4
64	33	15	18	19	14	36	61
12	21	59	38	39	58	24	9
	12		6		7		9
53	44	6	27	26	7	41	56
25	8	42	55	54	43	5	28
	8		10		11		5
40	57	23	10	11	22	60	37
20	13	35	62	63	34	16	17
	13		3		2		16
45	52	30	3	2	31	49	48

FIG. 17.

$2x2$ squares all have same sum

See Figs. 1, 2, 3, where $n=3$ and Fig. 13 where $n=7$.

In order to secure symmetry and balanced rows in the five center columns, especially when $n=5$, the following orders must be used:

	1	2	4	2	3	4	2	4	2	1
Above center cell	4	3,	1	3,	2	1,	3	1,	3	4
	1	2	4	2	4	3	2	4	2	1
Below center cell	4	3,	1	3,	1	2,	3	1,	3	4

See Figs. 7, 8 and 9.

17	19	31	29	137	139	142	144	61	63	51	49
	5		7		3 5		3 6		1 5		1 3
23	21	25	27	143	141	140	138	59	57	53	55
26	28	24	22	130	132	133	135	54	56	60	58
	8		6		3 4		3 3		1 4		1 6
32	30	18	20	136	134	131	129	52	50	62	64
97	99	111	109	73	75	78	80	45	47	35	33
2	5	2	7	1	9	2	0	1	1	9	
103	101	105	107	79	77	76	74	43	41	37	39
106	108	104	102	69	71	68	66	38	40	44	42
2	8	2	6	1	7	1	8	1	0	1	2
112	110	98	100	65	67	70	72	36	34	46	48
81	83	95	93	16	14	11	9	125	127	115	113
2	1	2	3		4		3	3	1	2	9
87	85	89	91	10	12	13	15	123	121	117	119
90	92	88	86	8	6	3	1	118	120	124	122
2	4	2	2		2		1	3	0	3	2
96	94	82	84	2	4	5	7	116	114	126	128

FIG. 18 d = *2* *12x12*

Odd and even groups alternate.

Since either $n-3$, or $n-5$, is an evenly even number, the remaining columns may be treated as when n is evenly even.

It is finally necessary to consider the special case of the group in the center cell of the given odd square. This cell presents the only insurmountable obstacle that stands in the way of constructing a completely symmetrical oddly even magic square.

17	18	28	27	137	138	143	144	59	60	50	49
	5		7		3 5		3 6		1 5		1 3
20	19	25	26	140	139	142	141	58	57	51	52
29	30	24	23	133	134	131	132	55	56	62	61
	8		6		3 4		3 3		1 4		1 6
32	31	21	22	136	135	130	129	54	53	63	64
97	98	108	107	73	74	79	80	43	44	34	33
2	5	2	7	1	9	2	0	1	1		9
100	99	105	106	76	75	78	77	42	41	35	36
109	110	104	103	68	67	70	69	39	40	46	45
2	8	2	6	1	7	1	8	1	0	1	2
112	111	101	102	65	66	71	72	38	37	47	48
81	82	92	91	16	15	10	9	123	124	114	113
2	1	2	3		1		3	3	1	2	9
84	83	89	90	13	14	11	12	122	121	115	116
93	94	88	87	8	7	2	1	119	120	126	125
2	4	2	2		2		1	3	0	3	2
96	95	85	86	5	6	3	4	118	117	127	128

FIG. 22

In every symmetric odd square the middle number, which is equal to half the sum of the first and last numbers in the series, must be in the center cell. Hence this middle number is really self-symmetrical both as to value and position. Therefore the middle group must always be in the center cell, and must always be self-symmetric That is, this group must be written in one of the Z orders. Furthermore, in order to balance the two middle rows under the scheme for

arranging the three, or five, central columns just explained, the two largest numbers of this group must be put in the upper row. This leaves the two middle columns of the required square out of balance. Therefore another group in the center column of the given square must be written in the reversed Z order. In all the following examples this group has been put in the bottom cell. The group in the center

$$\begin{matrix} 3 & 4 \\ 1 & 2 \end{matrix}$$

has been put in the order and that in the bottom cell

$$\begin{matrix} 4 & 3 \\ 2 & 1 \end{matrix}$$

Hence the middle row and the bottom row of the three and five center columns must be written in the orders:

Middle row $\begin{matrix} 1 & 3 & 3 & 4 & 3 & 1 \\ 4 & 2, & 1 & 2, & 2 & 4 \end{matrix}$

and $\begin{matrix} 1 & 2 & 4 & 2 & 3 & 4 & 2 & 4 & 2 & 1 \\ 4 & 3, & 1 & 3, & 1 & 2, & 3 & 1, & 3 & 4 \end{matrix}$

Bottom row $\begin{matrix} 1 & 3 & 4 & 3 & 3 & 1 \\ 4 & 2, & 2 & 1, & 2 & 4 \end{matrix}$

and $\begin{matrix} 1 & 2 & 4 & 2 & 4 & 3 & 2 & 4 & 2 & 1 \\ 4 & 3, & 1 & 3, & 2 & 1, & 3 & 1, & 3 & 4 \end{matrix}$

Since the group in the top cell of the given square has been written

$$\begin{matrix} 3 & 4 \\ 2 & 1 \end{matrix}$$

in the order , this makes the top numbers and the bottom numbers in the two middle columns symmetrical with respect to the horizontal axis, but not symmetrical with respect to the center. In all other respects the required square is symmetric.

74	106	554	570	234	202
	2		9		4
122	90	538	522	218	250
394	426	298	314	170	138
	7		5		3
442	410	266	282	154	186
330	362	58	42	490	458
	6		1		8
378	346	26	10	474	506

FIG. 23.

392	2184	904
1672	1160	648
1416	136	1928

FIG. 24.

The real purpose of this study was to construct, if possible, a perfect oddly even magic square. The conclusion is that an oddly even magic square can be constructed from any given symmetric odd square that shall be symmetrical except with respect to two pairs of numbers in the two middle columns. It seems doubtful whether a completely symmetric oddly even magic square is obtainable.*

*No better result could have been obtained by putting more groups in the Z order.

[EDITOR'S NOTE: A second installment of this paper will appear in an early issue of the News Letter.]

The Binomial Theorem

By IRBY C. NICHOLS

One who is making advanced preparation to teach High School mathematics has already acknowledged an appreciation of the fact that greater acquaintance with a given topic—its history, its development, its applications—gives greater inspiration and force in a classroom presentation of such a topic, even though it may happen that a lack of time or the advanced nature of this knowledge may prevent it from being presented in a *formal* way.

However, a topic is often inadequately treated simply because a too limited training relative to it has previously given a narrow view of its significance and has led the teacher to feel that it is "not very important after all". As a simple example of this attitude, consider the binomial theorem. Little attention is seemingly paid to this theorem except to indicate the expansion of a binomial $(a+b)^n$ for values of $n=1, 2, 3, 4$. It deserves more notice, for it has considerable significance and many ready practical applications. Its most familiar use is the expansion of the square, the cube, or the fourth power of some binomial. It is sometimes used to find roots of fractional powers of numbers when logarithms are not convenient. A less familiar, though very important, use is its application in solution of problems of finance where the quantity sought is the effective rate of interest.*

*Several articles by the present writer to illustrate this use of the Binomial Theorem have appeared in the News Letter. See January, 1932, and February-March, 1932.

Another nice use is the solution of problems in probability. Let p be the probability that an event will happen, q the probability of its failure to happen, and n the number of trials. Now consider the binomial expansion

$$(p+q)^n = p^n + np^{n-1}q + \frac{n(n-1)}{1.2}p^{n-2}q^2 + \frac{n(n-1)(n-2)}{1.2.3}p^{n-3}q^3 + \ldots\ldots + q^n.$$

The first term p^n is the probability that an event will happen n times in n trials. The second term is the probability that an event will happen $n-1$ times in n trials and fail once; since this one failure may occur in n ways in n trials, the total probability of the event happening as prescribed is $np^{n-1}q$. The third term is the probability that an event will happen $n-2$ times and fail twice in n trials; but in n trials, this situation can occur in $\frac{n(n-1)}{1.2}$ ways; hence $\frac{n(n-1)}{1.2}p^{n-2}q^2$ is the *total* probability of the event occurring as prescribed. And so correspondingly for all the terms of the binomial expansion. Hence the general theorem that the successive terms of the binomial expansion of $(p+q)^n$ give the respective probabilities that, in n trials, an event will happen exactly n times, $n-1$ times, ... 2, 1, 0 times.

Incidentally these corollaries may be noted:

(1) The sum of those terms of this same binomial series in which the exponent of p equals or exceeds r gives the probability that an event will happen at least r times in n trials.

(2) That it will happen at most r times is the sum of those terms in which the exponent of p is less than or equal to r.

(3) †And the probability that it will happen at least once is $1-q^n$.

Illustrations:

(1) What is the probability of obtaining *exactly* 4 aces in 5 throws with 1 die?

†For further information on the Point Binomial consult almost any advanced algebra, or text containing a chapter on Probability.

From the general theorem above we have $p=1/6$, $q=5/6$, $n=5$. Hence $(p+q)^n$ becomes here $(1/6+5/6)^5$. The question then is to find the value of the term containing the 4th power of p, which is $(5/1)\ (1/6)^4(5/6)^1 = 25/7776 = .003215$. Hence the probability of obtaining exactly 4 aces in 5 throws with one die is 3215 in 1,000,00 or about 32 in 10,000.

(2) What is the probability of obtaining exactly 4 aces in 5 throws with 2 dice?

Using 2 dice in 5 throws is equivalent to 10 throws with one die. Hence the immediate set-up $(2/12+10/12)^{10}$ to find the term

$$(10.9.8.7/1.2.3.4)\ (2/12)^4(10/12)^6,$$

whose value is .054, the probability involved.

Other illustrations may be cited, but these two are sufficient to make clear the point at issue. The writer feels that the force and attractiveness of this use of the binomial theorem are great enough to win the appreciation of all who are familiar with it, and cause those who teach High School mathematics to place more emphasis upon it in their courses in algebra.

---

*The Unit Assignment in Algebra and Geometry

LUCILLE M. BOSTICK
Sophie Wright High School
New Orleans, La.

This subject can hardly be developed without a consideration of the following problems:

I. What is meant by the unit assignment?

II. What is the value of the unit assignment as compared with the daily recitation plan?

III. Do algebra and geometry lend themselves to this form of procedure?

*This paper was published in the January News Letter of this Volume. Through an error for which the Editor was not responsible it was credited to Wm. A. Payne.
The Editor.

After a necessarily brief discussion of these topics, I shall quote a specimen assignment in algebra from a National Survey and also give my plan for an assignment in geometry.

The learning unit[1] as defined by William C. Ruediger is "any division of subject matter, large or small, that when mastered gives an insight into, an appreciation of, or a mastery over, some aspect of life." This term then would cover the unit assignment or any one of its subdivisions. In this discussion, however, the "unit" will be used to designate one of the subdivisions of the assignment rather than the whole.

The unit assignment, as applied to mathematics, is the teacher's plan for the pupils' activities and experiences which in his judgment are best fitted to produce the mastery of particular related units grouped under one general topic. It is usually in the form of a guide sheet, and consists of: (1) directions for study; (2) references; (3) a list of supplementary projects; (4) an outline of essentials (sometimes the maximum and sometimes the minimum); and (5) a tentative time allotment. There is often added in addition to the above, an introductory paragraph to stimulate interest and curiosity; necessary explanations; helps in the form of enumeration of special difficulties; additional optional work; and a test on the assignment. Some also add reteaching and retesting. The methods used in covering the assignment are left to the individual teacher.

A National Survey of Education[2] conducted in 1928 by the United States Department of the Interior under the direction of Dr. Leonard V. Koos, of the University of Chicago and at the suggestion of the North Central Association of Colleges and Secondary Schools, found that the unit assignment is being widely used all over the country, particularly in the larger schools and those of the reorganized type; and in practice, differentiated assignments, whether listed under the Project, Problem, Contract, Laboratory, Morrison, Dalton or Winnetka plans, were all characterized by the unit assignment.

The daily recitation plan emphasizes the teacher rather than pupil participation and dates back to the days when the usual lesson assignment was, "Take the next ten pages," or "Work from the first through the tenth examples on page 67." Such assignments did not stimulate interest and curiosity, which form the basis for intelligent and pur-

[1]Ruediger, William C.—The Learning Unit School Review 40, pp. 176-81, March '32.

[2]Bulletin 1932, No. 17; Monograph No. L3, p. 355, U. S. Department of the Interior.

poseful study. They were sufficient, perhaps, a generation ago when the high school pupils were a more or less highly selected group capable of making their own way, but with the heterogeneous grouping of today the assignments must provide for individual differences, guide, direct, and inspire. The unit assignment does all these, because by setting a definite problem it fosters pupil activity and at the same time is elastic enough for even the slow pupils to cover at least part of the minimum essentials, which encourages further effort on their part. Therefore it is psychologically sound.

Does this mean that all of our subject matter should be handled through the unit assignment? No, some units may best be handled by the daily recitation plan; such as, the axioms and definitions in Geometry, for instance. This calls for judgment on the part of the individual teacher.

An interesting experiment[3], to determine the comparative values of these two plans, was conducted in Queen Anne High School in Seattle, Washington, in 1928-29, during the fall semester. Four classes of unselected groups in General Science were used. They had the same teacher, used the same texts, same equipment, and in so far as was possible all the factors, except the method, were kept identical. Two groups followed the unit assignment, and two the daily recitation plan. Results were tabulated and compared according to matched partners corresponding in age, sex, I. Q., term grades and semesters in school. The study showed a slight general advantage in favor of the unit assignment plan, as seventeen made more progress than their matched partners in the daily recitation group. Neither yielded better results consistently, but varied with the type of subject matter. Factual knowledge was best taught by the unit assignment. Hence, the authority for my above statement that some ideas can best be handled by the old time method, as all do not lend themselves readily to the unit assignment.

The subject matter of mathematics, due to its logical sequence, lends itself very readily to the unit assignment. In fact, the text books in both algebra and geometry are in themselves guide sheets, as related units are already grouped under topic headings and the teacher can very easily round out the assignment. In the 362 schools studied in the above mentioned National Survey, about eighty per cent of their several offerings were presented by the unit assignment, and mathe-

[3]Shelton, A.—An Experimental Study of the Daily Recitation Versus the Unit Plan—Ed. Journal, Vol. 38, '30, pp. 694-699.

matics ranked fourth highest in the subject matter fields, averaging fifty-seven per cent; plane geometry, seventy-three; algebra, sixty-three; solid geometry, fifty-five; and trigonometry only thirty-five per cent. The assignments in these subjects, of course, are briefer than those in history and English.

I shall now quote a unit assignment in algebra which is adequate yet brief. It was submitted by the South Philadelphia High School for girls in Philadelphia, Pa., and published in the report of the same survey[4]:

Algebra I. Time: one week. Text: Durell and Arnold, Shorling-Clark-Lidell; Instructional Tests in Algebra. Fractions (all work in Durell and Arnold).

1. Study p. 160. Note that fractions in Algebra in general have the same properties as fractions in Arithmetic. Transformation of fractions.
2. Reduction of fractions to lowest terms. In exercises 86 and 87, pp. 161-62, work the odd numbered problems.
3. Reduction of an improper fraction to a mixed number. Study p. 163 and work examples 2, 4, 5, 7, 10, 12, 14, 20 and 22 in exercise 88.
4. Reduction of mixed expressions to a fraction. Study p. 163 and work examples 6, 9, 11, 14, 19 and 20 in exercise 89.
5. Do you remember how to do addition, subtraction, multiplication and division of whole numbers? Test yourself by working example 14, p. 44; example 37, p. 48; example 30, p. 82; example 14, p. 82; example 20, p. 87. Work exercises 28 and 30, p. 19.
6. Maximum assignment; Study pp. 176 and 177, Part II. Work examples 1-3, p. 178; example 1-3, p. 180; example 16-18, p. 180.

As far as Geometry is concerned, I should like to submit my own plan:

Geometry I. Time: one week. Text: Wells and Hart, Modern Plane Geometry; Avery, Geometry Workbook. Congruency of triangles.

[4]National Survey, Bulletin 1932, No. 17, Monograph No. 13, pp. 331-32; 355.

1. Study p. 25. Note: In proving triangles congruent, find first the equal sides. Study the figure to find any sides that may be equal by identity. If you can not find any equal sides, you cannot prove the triangles equal. Before your work is complete, you must be able to name the three parts of one triangle that are equal respectively to the three parts of the other.
2. Triangles having two sides and the included angle equal. Study pp. 26-28 and work examples 7, 10, 13, 19 and 20.
3. Triangles having two angles and the included side equal. Study p. 30 and the lower half of p. 31. Draw figures for exercises 30 and 34. Work exercises 31, 33 and 37. Complete statements on pp. 23 and 26 of work book.
4. Triangles having three sides equal. Study p. 35. Draw figures for exercises 50 and 52 on p. 36. Work exercises 47 and 51. Complete the statements on pp. 43 and 44 of the work book.
5. Answer questions of exercises 56-60 on p. 36 of text. Write the theorems for the exercises on pp. 43 and 44 of the work book.
6. Optional work: exercises 7-9 on p. 272 of the text.
7. Method test on pp. 45 and 46 of work book. (Regular trimester test to be taken later).

Present day educators attach great importance to the pupil's attitude towards his task, usually referred to as "interest" or "wholehearted, purposeful activity." Therefore it would appear highly desirable to know the pupils' reaction to the unit assignment. An attempt was made to discover the reaction of boys and girls of various levels of intelligence, accomplishment and application to this type of assignment. What do they consider the advantages or disadvantages of this plan of instruction as contrasted with the class room procedure of the daily recitation plan? The study[5] was conducted by Thomas W. Harvey of the Memorial High School, Planesville, Ohio. We are interested only in the results obtained by the inquiry form filled out by 75 pupils in three tenth-grade sections in plane Geometry. Forty-five distinct advantages of the unit assignment were

[5]Billet, Roy O.—High School Pupils' Opinion of the Unit Plan, School Review, Vol. 40: pp. 17-32, Jan. '32.

listed against ten disadvantages. It was the general concensus of opinion that the unit plan places the pupil on his own responsibility, he may work at his own rate of speed, is the only one to blame if he does not get a good mark, and "makes failure unnecessary for even a slow pupil if he is willing to work."

Since the unit assignment provides for individual differences, holds the interest of the pupils, and produces better results than the traditional daily recitation plan, it should be used more extensively in both algebra and geometry.

Book Review Department

Edited by
P. K. SMITH

Differential Equations, by A. Cohn, Johns Hopkins University. D. C. Heath and Company, 1933. Second edition, completely revised.

The first edition appearing in 1906 was printed on good thin paper and with marginal space cut down so that the small book looked like a brief course, but it proved to be as complete a treatment as given by Johnson or Murray. The second edition is printed on first quality paper of heavier texture and the pages are now $5\frac{1}{4}$ by $7\frac{3}{4}$ inches and have wider margins. The type is larger. The new book appears physically to be a full treatise on differential equations for a first course. The numbered pages total 337, the increase being 66 pages. The chapters are the same in number and in title but altered in arrangement. The chapter on total differential equations is the tenth rather than the sixth. The chapter on integration in series is moved forward from the eleventh to the ninth, thereby bringing the chapter on systems of equations closer to the one on partial differential equations.

In the second edition, this fine textbook has been completely revised. Some exercises have been written anew and more have been added to the lists. The practical applications have been increased in number and are more fully treated in the interpretations to the student. The forms of differential equations of the first order are rearranged in the best order, variables separable, homogeneous, not homogeneous but linear in x and y, being the first three forms in chapter two. The

chapter dealing with integration in series has been largely rewritten and much improved.

The author has well refrained from adding new chapters to his elementary treatise as he was tempted to do when he prepared the first edition. The suggestions which were made by Professor C. R. MacInnes in his review of the first edition have been heeded. In that review MacInnes commented: "Many interesting little historical notes have been inserted and references have also been added freely."

As student and teacher of the first edition, the present reviewer has found the textbook dependable as to selection of material, clear in the explanations, and largely free from errors in the printing and in the answers to the many exercises. The only mistake in bad grammar that has been marked stands corrected.

WILSON L. MISER,
Vanderbilt University.

March 9, 1934.

Inversive Geometry, by Frank Morley and F. V. Morley. Published by Ginn and Company, 1933; 273 pages, $3.50.

The many students of Dr. Frank Morley welcome with much pleasure this long looked for book, published jointly with his son. The style of writing is very concise, and most readers will find it quite thought-provoking.

The book is an introduction to algebraic geometry with particular reference to the operation of inversion. In part I, the authors go over the application of number to Euclid's planar geometry and arrive at the Euclidean group of operations. By adjoining an inversion the wider inversive group is treated. The geometry is now non-Euclidean. The authors have sought to apply Klein's views of a geometry to the inversive group. In the chapter on Flow, simple cases are given to serve to introduce conformal mapping and to emphasize the fact that the study of an analytic function is the study of a region. The final chapter of Part I is on Differential Inversive Geometry.

In Part II, applications are mainly considered. In Chapter XII, the map-equations of a line and a circle are given. In Chapter XIII, the construction of any regular polygon of $2n + 3$ sides is considered by tying knots in a strip of paper. In the next chapter, the equation of a planar motion is treated. In XVII, rational curves are taken up and methods for their mechanical construction are explained, "thus the general rational curve, after an inversion when necessary, can be mechanically described by two pins moving in two slots, the slots being

simpler than the proposed curve." A few of the other topics considered in Part II are the nine-point circle, conics, the cardioid and the deltoid, Cremona transformations, and Clifford's chain.

The book covers a wide field. The reviewer is impressed favorably and desires to express his conviction that this book should be read by all students of pure mathematics.

HENRY A. ROBINSON,
Agnes Scott College,
Decatur, Ga.

Problem Department

Edited by
T. A. BICKERSTAFF

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

PROBLEMS FOR SOLUTION

No. 57. Proposed by R. B. Thompson, Beaver Crossing, Nebraska.

Determine a point such that the sum of the nth powers of the distances to three fixed points is a minimum.

No. 58. Proposed by R. B. Thompson, Beaver Crossing, Nebraska.

Find a type of determinant that will repeat its form under multiplication and addition and when expanded will be the sum of n squares.

No. 59. Proposed by A. W. Randall, Prairie View, Texas.

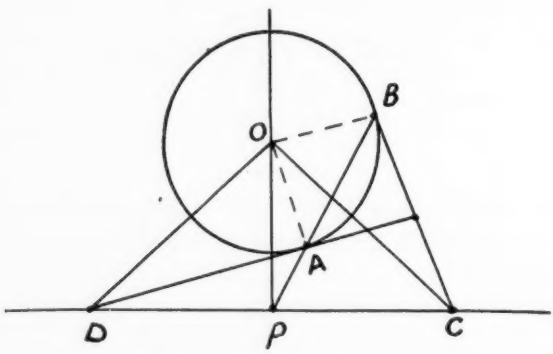
Find the volume of a 2-inch square hole cut through a sphere 12 inches in diameter, the axis of the whole being the diameter of the sphere.

SOLUTIONS

No. 52. Proposed by H. T. R. Aude, Colgate University.

Given a circle with the center at O and a point P not on the circle. A line is drawn through P cutting the circle in two points at which the tangents are drawn. These meet the line through P which is perpendicular to PO in the points C and D. Are PC and PD equal in length?

Solved by Henry Schroeder, Louisiana Polytechnic Institute.



In the above figure PAB is a secant, DA and CB are tangents and DC is perpendicular to OP.

Points O,P,B,C, are concyclic.
Hence angle OPB equals angle OCB.

Points D,P,A,O are concyclic.
Hence angle OPB equals angle ODA.

Therefore angle OCB equals angle ODA.

Right triangles CCB and ODA are congruent.

OD equals OC.

OP is perpendicular bisector of CD.

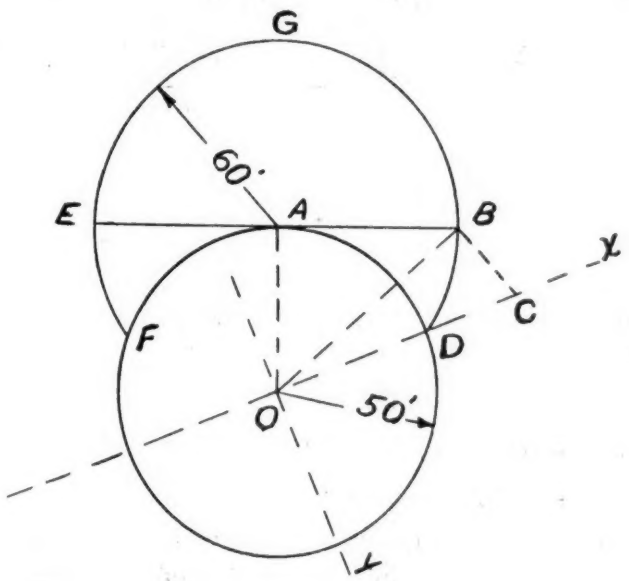
PD equals PC.

Also solved by A. W. Randall, Prairie View State College, Texas.

No. 54. Proposed by H. T. R. Aude, Colgate University

If a horse is tied by a rope 60 feet long, staked at the side of a vertical cylindral tank of 100 feet diameter, find the area over which the horse can graze.

Solved by Buford E. Gatewood, Louisiana Polytechnic Institute.



EF = BD = arc of involute of circle

Equations of involute:

$$X = r(\cos\theta + \theta \sin\theta)$$

$$Y = r(\sin\theta - \theta \cos\theta)$$

AD = 60′ = 6/5 radians = 68°45′18″

X = OC = 50(cos 68°45′18″ × 6/5 sin 68°45′48″)

OC = 74.04′

BC = Y = 50(sin 68°45′18″ − 6/5 cos 68°45′18″)

BC = 24.86′

$$\text{Area required} = \text{Area ABGEA} + \text{Area ABDA} + \text{Area AEFA}$$

$$= \text{Area ABGEA} + 2\ \text{Area ABDA}$$

$$= \text{Area ABGEA} + 2\ (\text{Area ABCOA} - \text{Area ADOA} - \text{Area BCDB})$$

$$= \text{Area ABGEA} + 2(\text{Area ABOA} + \text{Area BCOB} - \text{Area ADOA} - \text{Area BCDB})$$

$$= \frac{\Pi(60)^2}{2} + 2\left|\frac{50\times 60}{2} + \frac{74.04\times 24.86}{2} - \frac{6}{5}\times\frac{\Pi(50)^2}{2\Pi} - \int_{50}^{74.04} y\,dx\right|$$

$$\int_{50}^{74.04} y\,dx = \int_{0}^{\frac{6}{5}} 50(\sin\theta - \theta\cos\theta)(50\theta\cos\theta)\,d\theta = 2500\int_{0}^{\frac{6}{5}\text{ rad.}} (\theta\sin\theta\cos\theta - \theta^2\cos^2\theta)\,d\theta$$

$$= 200.37 \text{ sq. ft.}$$

$$\text{Area} = 5658.88 + 2(1500 + 920.32 - 1500 - 200.37)$$

$$= 7098.78 \text{ square feet.}$$

Also solved by A. W. Randall, Prairie View State College, Prairie View, Texas, and by J. W. Ault, Cedarville College, Cedarville, Ohio.

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